



Semnan University
Faculty of Mechanical Engineering

دانشکده مهندسی مکانیک

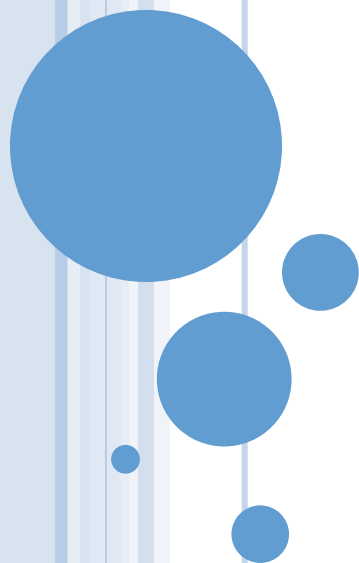


دانشکده مهندسی مکانیک

درس طراحی سیستم های شاسی خودرو

VEHICLE CHASSIS SYSTEMS DESIGN

*Chapter 6 – Cornering
Class Lecture*



❑ **CONTENTS:**

- ❖ Chapter 1: Introduction
- ❖ Chapter 2: Accelerating Performance
- ❖ Chapter 3: Braking Performance
- ❖ Chapter 4: Road Loads
- ❖ Chapter 5: Ride
- ❖ Chapter 6: **Cornering**
- ❖ Chapter 7: Suspension
- ❖ Chapter 8: Steering System
- ❖ Chapter 9: Roll-over

INTRODUCTION

- ❑ The cornering behavior of a motor vehicle is an important performance mode often equated with handling.
- ❑ "Handling" is a loosely used term meant to imply the responsiveness of a vehicle to driver input, or the ease of control.
- ❑ As such, handling is an overall measure of the vehicle-driver combination. The driver and vehicle is a "closed-loop".
- ❑ For purposes of characterizing only the vehicle, "open-loop" behavior is used.



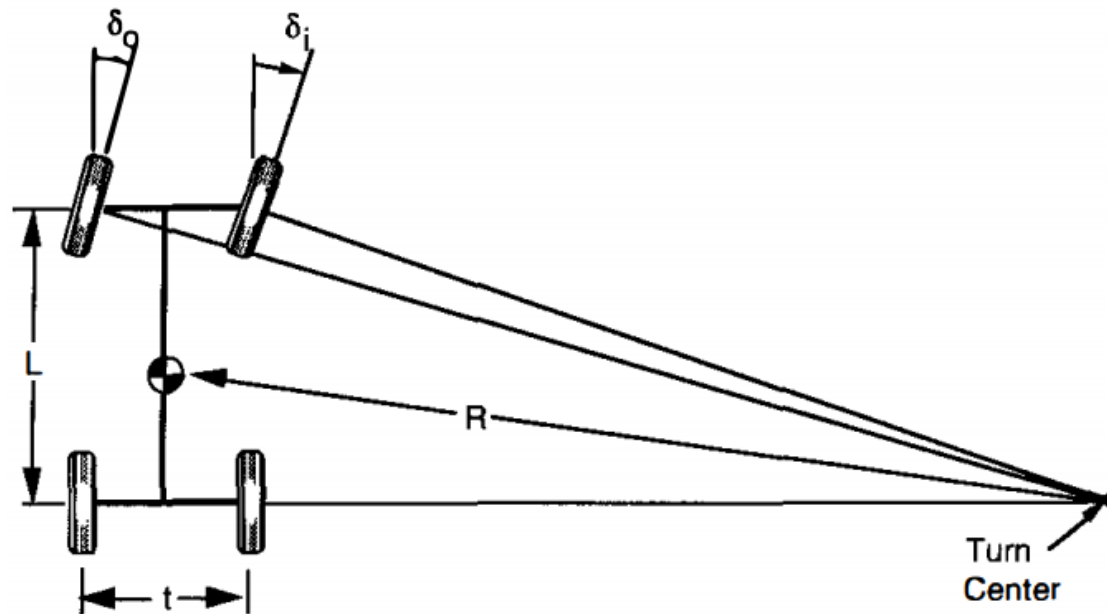
INTRODUCTION

- ❑ Open loop refers to vehicle response to specific steering inputs, and is more precisely defined as "directional response" behavior.
- ❑ The most commonly used measure of open-loop response is the understeer gradient (a measure of performance under steady-state conditions).
- ❑ We first analyze turning behavior at low speed, and then consider the differences that arise under high-speed conditions.



LOW-SPEED TURNING

- At low speed (parking lot maneuvers) the tires need not develop lateral forces. Thus they roll with no slip angle.
- Ackerman Geometry:



LOW-SPEED TURNING

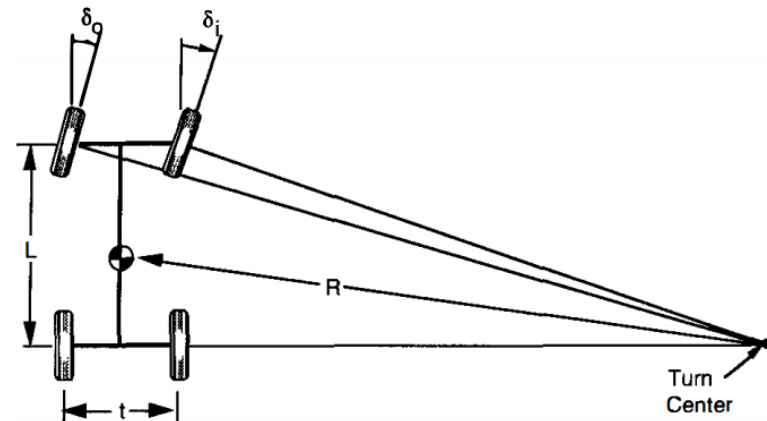
□ Center of Turn:

- ❖ The perpendicular from each wheel should pass through the same point (the center of turn)
- ❖ If they do not pass through the same point, the front tires will "fight" each other in the turn, with each experiencing some scrub (sideslip) in the turn.

□ The ideal turning angles:

$$\delta_o \equiv \frac{L}{(R + t/2)}$$

$$\delta_i \equiv \frac{L}{(R - t/2)}$$



□ The average angle of the front wheels: the Ackerman Angle

$$\delta = L/R$$

LOW-SPEED TURNING

- ❑ Errors, or deviations, from the Ackerman in the left-right steer angles can have a significant influence on front tire wear.
- ❑ Errors do not have a significant influence on directional response; however, they do affect the centering torques in the steering system.
- ❑ With correct Ackerman geometry, the steering torques tend to increase consistently with steer angle, thus providing the driver with a natural feel in the feedback through the steering wheel.

LOW-SPEED TURNING

- The rear wheels off-tracking distance, Δ

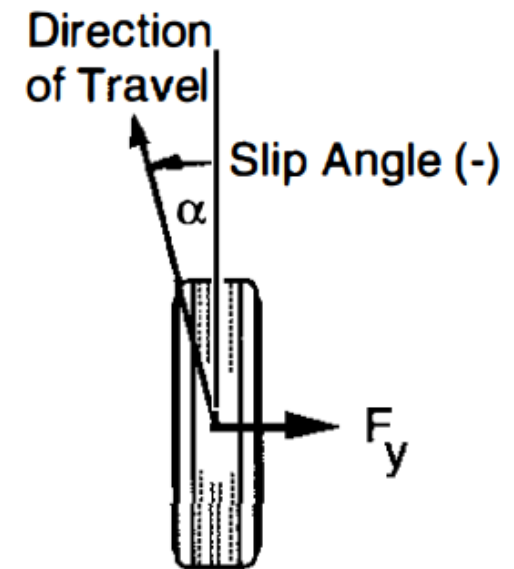
$$\Delta = R[1 - \cos(L/R)]$$

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} \dots \longrightarrow \Delta \cong \frac{L^2}{2R}$$

- ❖ off-tracking is primarily of concern with long wheelbase vehicles such as trucks and buses.

HIGH-SPEED CORNERING

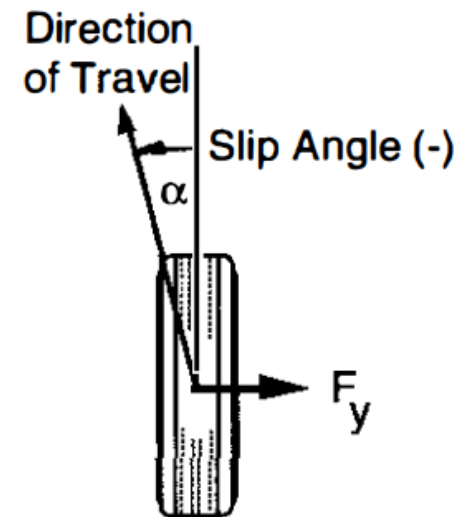
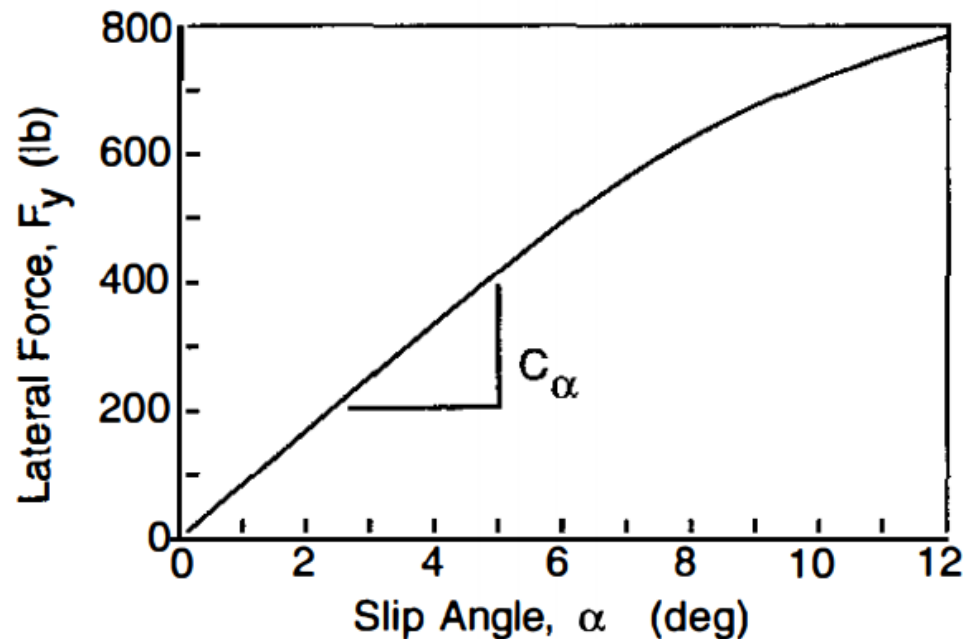
- ❑ At high speed, the turning equations differ because lateral acceleration will be present.
- ❑ To counteract the lateral acceleration the tires must develop lateral forces, and slip angles will be present at each wheel.
- ❑ The angle between its direction of heading and its direction of travel is known as slip angle, α .



HIGH-SPEED CORNERING

□ Tire Cornering Forces

- ❖ The lateral force, denoted by F_y , is called the “cornering force”.



HIGH-SPEED CORNERING

□ Tire Cornering Forces

- ❖ At low slip angles (5 degrees or less) the relationship is linear:

$$\longrightarrow F_y = C_\alpha \alpha$$

- ✓ C_α : Cornering Stiffness

- ❖ The cornering stiffness is dependent on many variables:

- ✓ Tire size and Type (radial- versus bias-ply construction), Number of plies, Cord angles, wheel width, tread, The load and inflation

- ❖ Because of the strong dependence of cornering force on load, tire cornering properties may also be described by the “cornering coefficient”: (Typ.=0.2)

$$\longrightarrow CC_\alpha = C_\alpha / F_z$$

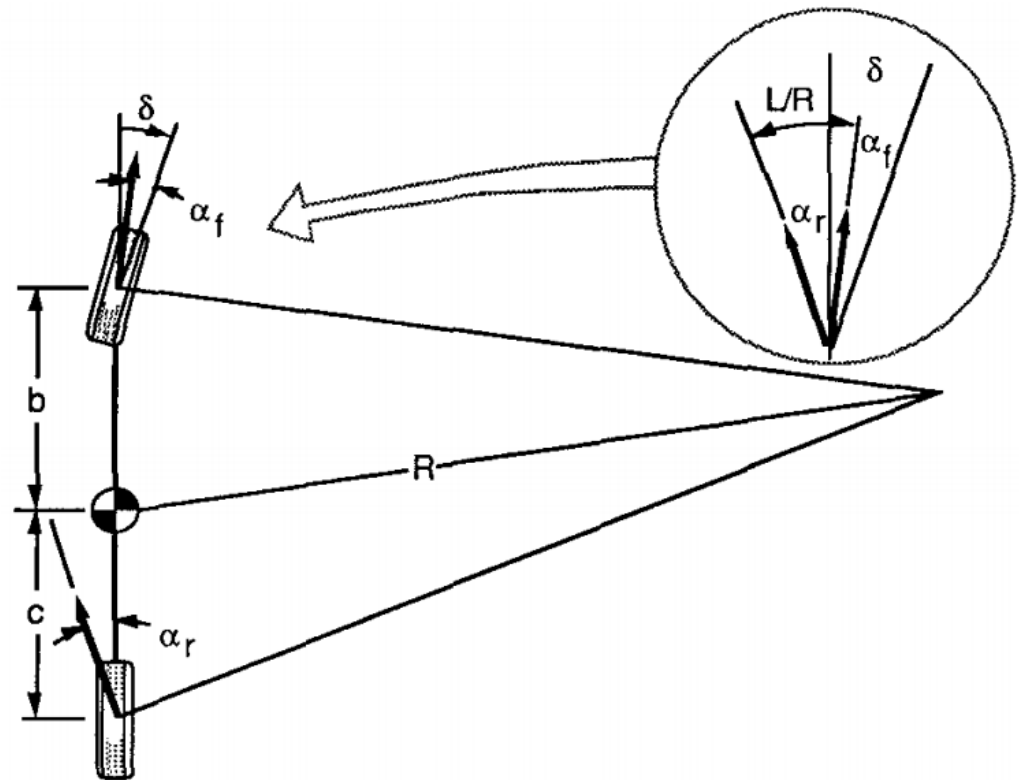


HIGH-SPEED CORNERING

❑ Cornering Equations

❖ The Bicycle Model

- ✓ At high speeds the radius of turn is much larger than the wheelbase of the vehicle.
- ✓ Then small angles can be assumed, and the difference between steer angles on the outside and inside front wheels is negligible.



HIGH-SPEED CORNERING

❑ Cornering Equations

- ❖ Sum of the forces in the lateral direction

$$\Sigma F_y = F_{yf} + F_{yr} = M V^2/R$$

F_{yf} = Lateral (cornering) force at the front axle

F_{yr} = Lateral (cornering) force at the rear axle

M = Mass of the vehicle

V = Forward velocity

R = Radius of the turn

- ❖ Moment equilibrium about the center of gravity

$$F_{yf} b - F_{yr} c = 0 \quad \longrightarrow \quad F_{yf} = F_{yr} c/b$$



HIGH-SPEED CORNERING

□ Cornering Equations

$$\longrightarrow M V^2/R = F_{yr} (c/b + 1) = F_{yr} (b + c)/b = F_{yr} L/b$$

$$\longrightarrow F_{yr} = M b/L (V^2/R)$$

- ✓ Proportional to the vehicle mass carried on the rear axle and lateral acceleration.
- ✓ Same for front axle

HIGH-SPEED CORNERING

❑ Cornering Equations

❖ The Slip Angles at the front and rear wheels:

$$\alpha_f = W_f V^2 / (C_{\alpha f} g R)$$

$$\alpha_r = W_r V^2 / (C_{\alpha r} g R)$$

❖ From the geometry of the vehicle in the turn:

$$\delta = 57.3 L/R + \alpha_f - \alpha_r$$

$$\rightarrow \delta = 57.3 \frac{L}{R} + \frac{W_f V^2}{C_{\alpha f} g R} - \frac{W_r V^2}{C_{\alpha r} g R}$$



HIGH-SPEED CORNERING

□ Cornering Equations

$$\rightarrow \delta = 57.3 \frac{L}{R} + \left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right) \frac{V^2}{g R}$$

δ = Steer angle at the front wheels (deg)

L = Wheelbase (ft)

R = Radius of turn (ft)

V = Forward speed (ft/sec)

g = Gravitational acceleration constant = 32.2 ft/sec²

W_f = Load on the front axle (lb)

W_r = Load on the rear axle (lb)

$C_{\alpha f}$ = Cornering stiffness of the front tires (lb_y/deg)

$C_{\alpha r}$ = Cornering stiffness of the rear tires (lb_y/deg)



HIGH-SPEED CORNERING

□ Understeer Gradient

❖ The equation in a shorthand form:

$$\delta = 57.3 L/R + K a_y$$

K = Understeer gradient (deg/g)

a_y = Lateral acceleration (g)

✓ It describes how the steer angle of the vehicle must be changed with the radius of turn or the lateral acceleration.

✓ Understeer Gradient: $K = \left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right)$



HIGH-SPEED CORNERING

□ Understeer Gradient $\longrightarrow \delta = 57.3 L/R + K a_y$

❖ 1) Neutral Steer:

$$W_f / C_{\alpha f} = W_r / C_{\alpha r} \rightarrow K = 0 \rightarrow \alpha_f = \alpha_r$$

- ✓ On a constant-radius turn, no change in steer angle will be required as the speed is varied.
- ✓ Steer angle required to make the turn will be equivalent to the Ackerman Angle.
- ✓ Physically, the neutral steer case corresponds to a balance on the vehicle such that the "force" of the lateral acceleration at the CG causes an identical increase in slip angle at both the front and rear wheels



HIGH-SPEED CORNERING

□ Understeer Gradient $\longrightarrow \delta = 57.3 L/R + K a_y$

❖ 2) Understeer:

$$W_f / C_{\alpha f} > W_r / C_{\alpha r} \rightarrow K > 0 \rightarrow \alpha_f > \alpha_r$$

- ✓ On a constant-radius turn, the steer angle will have to increase with speed.
- ✓ It increases linearly with the lateral acceleration and with the square of the speed.
- ✓ In the understeer case, the lateral acceleration at the CG causes the front wheels to slip sideways to a greater extent than at the rear wheels. Thus to develop the lateral force at the front wheels necessary to maintain the radius of turn, the front wheels must be steered to a greater angle.



HIGH-SPEED CORNERING

□ Understeer Gradient $\longrightarrow \delta = 57.3 L/R + K a_y$

❖ 3) Oversteer:

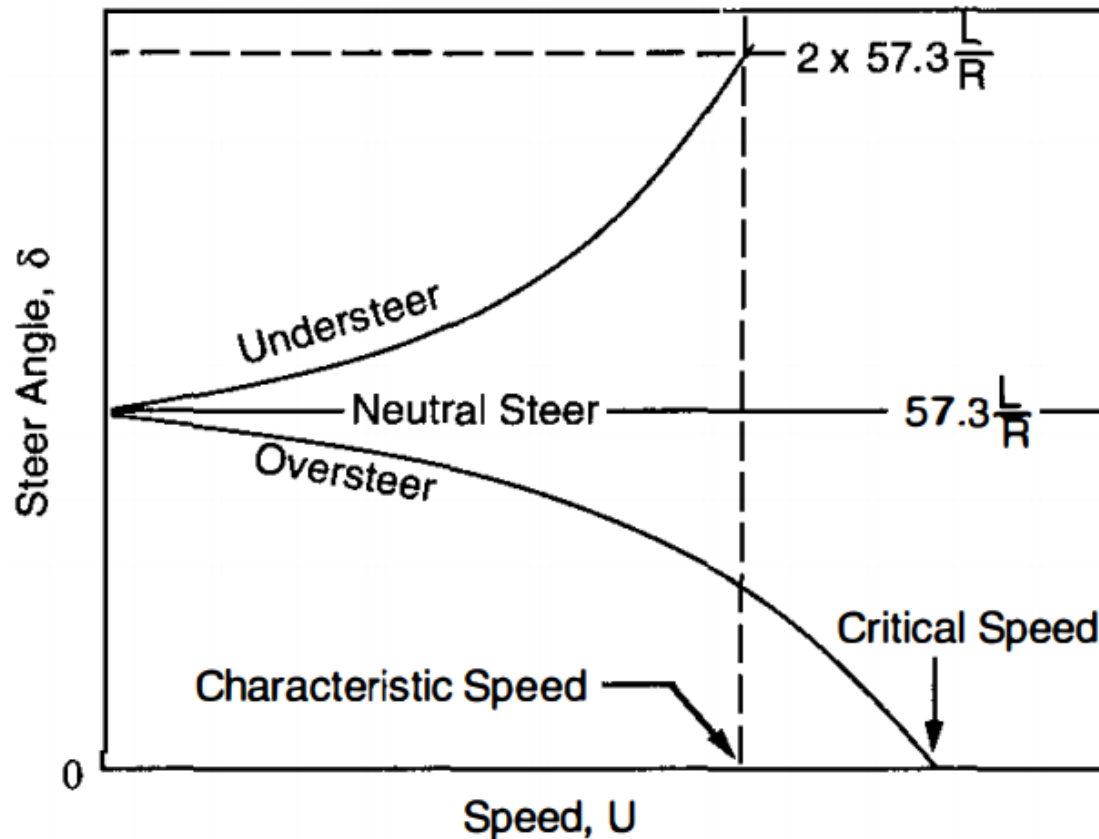
$$W_f / C_{\alpha f} < W_r / C_{\alpha r} \rightarrow K < 0 \rightarrow \alpha_f < \alpha_r$$

- ✓ On a constant-radius turn, the steer angle will have to decrease as the speed (and lateral acceleration) is increased.
- ✓ In this case, the lateral acceleration at the CG causes the slip angle on the rear wheels to increase more than at the front. The outward drift at the rear of the vehicle turns the front wheels inward, thus diminishing the radius of turn.
- ✓ The increase in lateral acceleration that follows causes the rear to drift out even further and the process continues unless the steer angle is reduced to maintain the radius of turn.

HIGH-SPEED CORNERING

Understeer Gradient

- ❖ The steer angle changes with speed on a constant-radius turn



HIGH-SPEED CORNERING

□ Lateral Acceleration Gain

- ❖ The purpose for steering a vehicle is to produce lateral acceleration.
- ❖ Ratio of lateral acceleration to the steering angle

$$\rightarrow \frac{a_y}{\delta} = \frac{\frac{V^2}{57.3 L g}}{1 + \frac{K V^2}{57.3 L g}} \quad (\text{deg/sec})$$

- ✓ When K is zero (neutral steer), the lateral acceleration gain is determined only by the numerator and is directly proportional to speed squared.
- ✓ When K is positive (understeer), the gain is diminished by the second term in the denominator, and is always less than that of a neutral steer vehicle.

HIGH-SPEED CORNERING

□ Lateral Acceleration Gain

$$\rightarrow \frac{a_y}{\delta} = \frac{\frac{V^2}{57.3 L g}}{1 + \frac{K V^2}{57.3 L g}} \quad (\text{deg/sec})$$

- ✓ When K is negative (oversteer), the second term in the denominator subtracts from 1, increasing the lateral acceleration gain.
- ✓ The magnitude of the term is dependent on the square of the speed, and goes to the value of 1 when the speed reaches the critical speed.
- ✓ Thus the critical speed corresponds to the denominator becoming zero (infinite gain) in the above equation.

HIGH-SPEED CORNERING

□ Yaw Velocity Gain

- ❖ A second reason for steering a vehicle is to change the heading angle by developing a yaw velocity (sometimes called "yaw rate").
- ❖ The yaw velocity, r :

$$r = 57.3 V/R \quad (\text{deg/sec})$$

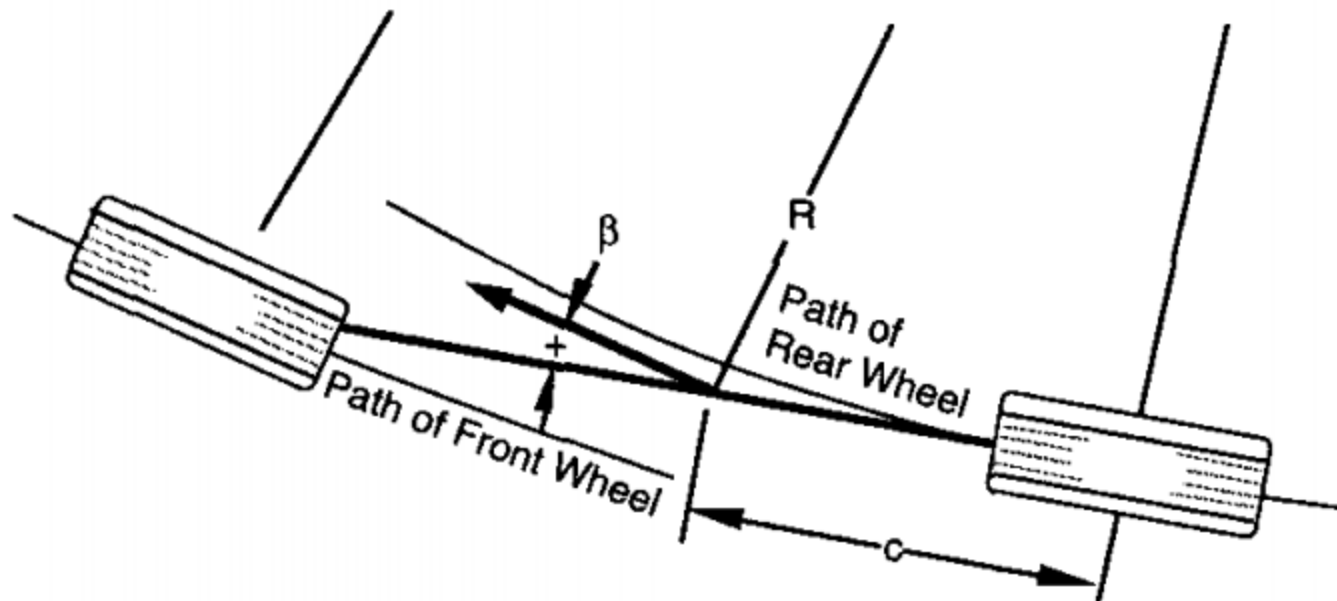
- ❖ The ratio of yaw velocity to steering angle:

$$\rightarrow \frac{r}{\delta} = \frac{V/L}{1 + \frac{K V^2}{57.3 L g}}$$

HIGH-SPEED CORNERING

❑ Sideslip Angle

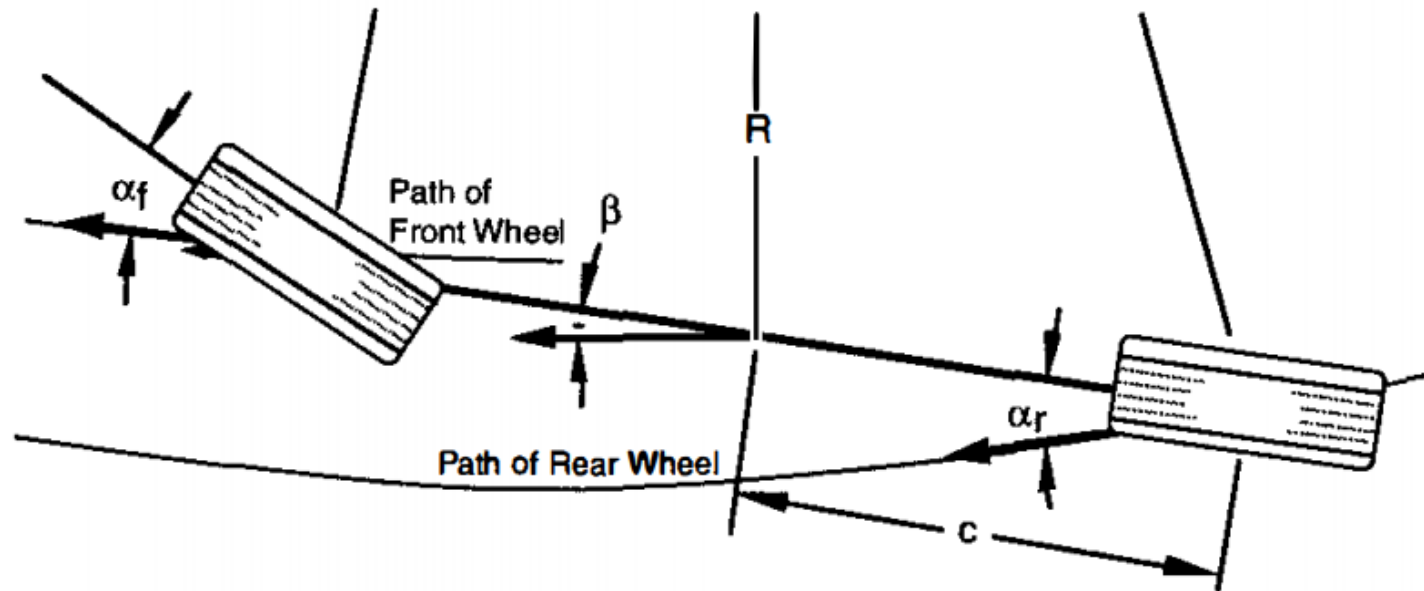
❖ At low speed turn



HIGH-SPEED CORNERING

❑ Sideslip Angle

❖ At high speed turn



HIGH-SPEED CORNERING

❑ Sideslip Angle

- ❖ For any speed the sideslip angle, β at the CG will be:

$$\begin{aligned}\beta &= 57.3 \, c/R - \alpha_r \\ &= 57.3 \, c/R - W_r \, V^2 / (C_{\alpha r} \, g \, R)\end{aligned}$$

- ❖ The speed at which the sideslip angle becomes zero:

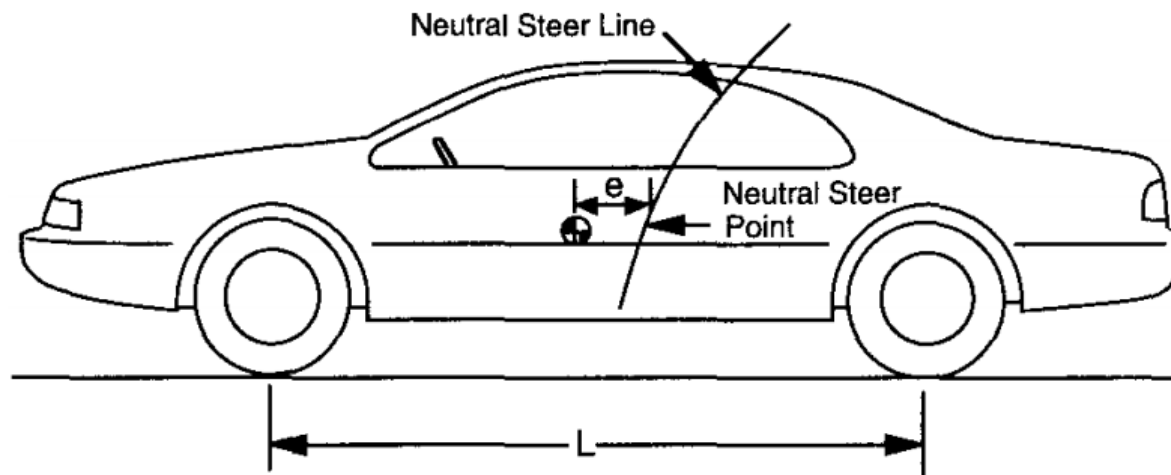
$$V_{\beta=0} = \sqrt{57.3 \, g \, c \, C_{\alpha r} / W_r}$$

- ✓ It is independent of the radius of turn.

HIGH-SPEED CORNERING

❑ Static Margin

- ❖ A term often used in discussions of handling is the static margin and, like understeer coefficient or characteristic speed, provides a measure of the steady state handling behavior.
- ❖ Static margin is determined by the point on the vehicle where a side force will produce no steady-state yaw velocity (i.e., the neutral steer point).



HIGH-SPEED CORNERING

□ Static Margin

- ❖ The static margin is defined as the distance the neutral steer point falls behind the CG, normalized by the wheelbase:

$$\text{Static Margin} = e/L$$

✓ Behind CG	→	Static margin is positive	→	Understeer
✓ At the CG	→	Static margin is zero	→	Neutral steer
✓ Ahead of CG	→	Static margin is negative	→	Oversteer.

- ❖ On typical vehicles the static margin falls in the range of 0.05 to 0.07 behind the CG.



SUSPENSION EFFECTS ON CORNERING

- ❑ Although tire cornering stiffness was used as the basis for developing the equations for understeer/oversteer, there are multiple factors in vehicle design that may influence the cornering forces developed in the presence of a lateral acceleration.
- ❑ Any design factor that influences the cornering force developed at a wheel will have a direct effect on directional response.
- ❑ The suspensions and steering system are the primary sources of these influences.

SUMMARY OF UNDERSTEER EFFECTS

UNDERSTEER COMPONENT	SOURCE
$K_{\text{tires}} = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}$	Tire cornering stiffness
$K_{\text{camber}} = \left(\frac{C_{\gamma f}}{C_{\alpha f}} \frac{\partial \gamma_f}{\partial \phi} - \frac{C_{\gamma r}}{C_{\alpha r}} \frac{\partial \gamma_r}{\partial \phi} \right) \frac{\partial \phi}{\partial a_y}$	Camber thrust
$K_{\text{roll steer}} = (\epsilon_f - \epsilon_r) d\phi/da_y$	Roll steer
$K_{\text{lfc s}} = A_f W_f - A_r W_r$	Lateral force compliance steer
$K_{\text{at}} = W \frac{p}{L} \frac{C_{\alpha f} + C_{\alpha r}}{C_{\alpha f} C_{\alpha r}}$	Aligning torque
$K_{\text{lt}} = \frac{W_f}{C_{\alpha f}} \frac{2 b \Delta F_{zf}^2}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \frac{2 b \Delta F_{zr}^2}{C_{\alpha r}}$	Lateral load transfer
$K_{\text{strg}} = W_f \frac{r v + p}{K_{ss}}$	Steering system

EXPERIMENTAL MEASUREMENT OF UNDERSTEER GRADIENT

□ Understeer gradient is defined by the SAE as:

- ❖ "The quantity obtained by subtracting the Ackerman steer angle gradient from the ratio of the steering wheel angle gradient to the overall steering ratio"

$$\rightarrow \delta = 57.3 L/R + K a_y$$

□ Experimental Measurement

- ❖ Constant Radius Method
- ❖ Constant Speed Method

